Geometry
Chapter 12

Volume

Surface Area

Similar shapes ratio area & volume
# Chapter 12: Lateral Area, Surface Area, and Volume

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Chapter 12 - Formulas

Note: The formulas with stars (**) next to them ARE given to you on all assessments!

### Prism

- \( LA = ph \) OR \( LA = \text{sum area of lateral faces} \)
- \( SA = LA + 2B \)
- \( V = Bh \)

### Pyramid

- \( LA = \frac{1}{2} pl \) OR \( LA = \text{sum area of lateral faces} \)
- \( SA = LA + B \)
- \( ** V = \frac{1}{3} Bh \)

### Cylinder

- \( ** LA = 2\pi rh \)
- \( SA = LA + 2B \)
- \( ** V = Bh \)

### Cone

- \( ** LA = \pi rl \)
- \( SA = LA + B \)
- \( ** V = \frac{1}{3} Bh \)

### Sphere

- \( ** A = 4\pi r^2 \)
- \( ** V = \frac{4}{3} \pi r^3 \)

### Hemisphere

- \( LA = 2\pi r^2 \)
- \( SA = 3\pi r^2 \)
- \( V = \frac{2}{3} \pi r^3 \)
Prism: Section 12-1

Define the following terms. Draw a prism and label each term in the diagram.

Prism:  

Diagram

Base(s):

Altitude:

Lateral Faces:

Lateral Edges:
What is the difference between a right prism and an oblique prism? Draw a diagram.

What is the difference between lateral area and total area?

What is a cube? Draw a picture.
Pyramid: Section 12-2

Define the following terms. Draw a pyramid and label each term in the diagram.

Pyramid:                  Diagram

Vertex:

Slant Height:

Label the following the diagram: Base, Altitude, Lateral Faces, and Lateral Edges
What is a regular pyramid?

What is the difference between an altitude (height) and a slant height?

**Cones and Cylinders: Section 12-3**

Diagram of Cylinder:  
Diagram of Cone:

Label the height and radius in both diagrams. Label the slant height of the cone.

How are the prism and cylinder similar? What is the difference?

How are the pyramid and cone similar? What is the difference?
Spheres: Section 12-4

What is a sphere? Draw a diagram?

Ratio of Areas and Volumes of Similar Solids: Section 12-5

If the scale factor of two similar solids is a:b, then...

the ratio of corresponding perimeters is __________.

the ratio of the base areas, lateral areas, and total areas of the solids is ___.

the ratio of the volumes of the solids is __________.
Surface Area and Volume WS1

Find the lateral area and total surface area for each shape. Draw each shape (or the side shapes) and organize work neatly!

1. Rectangular Prism with sides 4 X 4 X 9 in.
2. Cube with side 3.5 m
3. Right square pyramid with base sides of 18 feet and a height of 12 feet
4. Square prism with the base having sides of 5 cm; prism height of 8 cm
5. Regular square pyramid with base edge 24 and lateral edge 20
6. Regular triangular pyramid with base edge 4 and slant height 12 (exact value)
7. Right triangular prism with the base triangle having sides of 6.5, 7, and 10.5 and height of 4 to the side of 10.5; height of the prism is 15
8. Right prism with an isos. triangle base with sides of 13, 13, and 10; height of the prism is 7
9. Regular triangular pyramid with base edge 18 and height 13
10. Right square pyramid with base edge 14; pyramid height of 7 (exact value)
11. Right prism with equilateral triangle base with sides of 8; ht. of the prism is 13 (exact value)
12. Right trapezoidal prism which has an isosceles trapezoid base with parallel sides of length 6 cm and 12 cm and with legs of length 5 cm; the height of the prism is 10 cm
13. All six edges of a regular triangular pyramid have a length of 4. (exact value)
14. The diagram shows the plans for building a storage shed.

If the storage shed is made from aluminum panels, how many square feet of aluminum are needed to build the structure? Consider exterior panels only (not the floor or upper base of the rectangular prism) and round the answer to the nearest tenth.

15. How much material would be needed to cover a pillow of the shape below? All lengths are stated in inches rounded to the nearest tenth of an inch.
Find the volume of each figure. Round answers to the nearest tenth if necessary.

16. Regular right pentagonal pyramid with side 4 in., and height 15 in.
17. Right triangular prism with right triangle base of sides 6, 8, and 10 m, and a height of 12 m.
18. Cube with side length of 6 cm.
19. Rectangular prism with sides of 3 in, 4 in, and 15 in.
20. Find the length of a cube with a volume of 2197 in$^3$.
21. What is the height of a pyramid if it has a volume of 30 units$^3$ and has a base measuring 5 units by 6 units?
22. 
Surface Area and Volume WS2

Cylinder, Cone, Sphere

Find the surface area for each shape described. Draw a diagram and show work neatly organized. State answers in terms of \( \pi \).

1. Sphere with radius 10 inches
2. Sphere with a diameter 42 meters
3. Cylinder with a radius of 8 cm and a height of 11 cm
4. Cylinder with a radius of 17 and a height of 5
5. Cone with radius 4 feet and height 3 feet
6. Cone with a radius of 12 and a slant height of 13

Round answers to the nearest tenth when necessary.

7. What is the diameter of a sphere whose surface area is 113.04 m\(^2\)?
8. What is the surface area of a cylinder that has a diameter of 10 feet and a height of 8 feet?
9. A right cone has a radius of 5 inches and a surface area of 180 in\(^2\). What is the slant height of the cone?
10. Find the diameter of the base of a cylinder whose surface area equals 272 \( \pi \) m\(^2\) and height is 9 m.
11. Find the slant height of a cone with surface area of 178 \( \pi \) in.\(^2\) if the radius of the base is 4 in.

12. How much steel would be required to make a tuna can with a radius 2 in. and a height of 1.5 in.? (nearest tenth)
13. The diameter of a baseball is about 3.8 inches. How much leather is needed to cover the baseball? (exact & rounded nearest hundredth)
14. The radius of the Earth is about 7,900 miles. About 70% of the Earth's surface is water. How much of the Earth's surface is land? Round your answer to the nearest million square miles.

15. A roof in the shape of a half-cylinder covers the Center Street Ice Rink. The ice rink is a rectangle having a width of 50 feet and a length of 80 feet. What is the surface area, to the nearest square foot, of the roof?

16. A standard drinking straw is 19.5 cm long and has a diameter of 0.6 cm. How many square centimeters, to the nearest square centimeter, of plastic are needed to make 1,000 straws?

17. The exterior nose cone of the rocket shown is to be painted with a special sealant. How much surface area, to the nearest tenth of a square foot, needs to be covered with sealant?

18. The gel caplet is made to be filled with medicine. If the dimensions indicated are in millimeters, how many square millimeters of the "gel" coating are needed to make 100 caplets? (nearest sq. mm)
19. The shapes below represent kids toys that “pop” the end off. If the dimensions are indicated in centimeters, find the number of square centimeters (exterior surfaces only) that need to be painted for each toy. (nearest tenth)

Find the volume of each figure. Round answers to the nearest tenth if necessary; but leave in terms of $\pi$ if possible.

20. $r = 1.2 \text{ ft.}$

21.

22. Cylinder with a diameter of 8 cm and a height of 6 cm

23. Sphere with a diameter 16 inches

24. $r = 3.2 \text{ ft.}$

25.

26. What is the radius of a cone whose volume is $1500\pi$ cubic units$^3$ and whose height is 20 inches?

27. What is the height of a cylinder whose base has a circumference of $8\pi$ units and whose volume is $144\pi$ cubic units?
28. Find the volume, in terms of $x$, of a sphere with a diameter of $4x$.

29. What is the volume of the largest ball that will fit into a cubical box with edges of 12 inches? (round to the nearest tenth)

30. How many cubic inches of foam filler would fit around the ball placed in the box described in question 17? (nearest cubic inch)

31. Find the volume of the solid in the diagram below, which consists of a square prism with a cylindrical hole cut through the middle. (nearest tenth)
Find the total surface area of each figure. If necessary, state the answer in terms of $\pi$.

1. [Diagram of a cone with base radius 12 and height 15]

2. [Diagram of a cylinder with radius 5 and height 2]

3. [Diagram of a pyramid with base edge 12, side edge 8, and height 6]

4. [Diagram of a cone with slant height 41 and height 9]
7. Find the surface area of a cube whose side has a length of 4 cm.

8. Find the surface area of the rectangular prism whose dimensions are 4 x 16 x 22.

9. Find the surface area of a right square prism with a height of 6 inches. The sides of the base measure 2 inches.

10. Find the surface area of a prism whose base is a right triangle with sides of lengths 9 ft, 12 ft, and 15 ft and whose height is 10 feet.

11. Consider a sphere with a radius of 16 cm. If the radius of the sphere is doubled, what happens to the surface area? [Hint: try it!]

12. Find the surface area of a right cone with a radius 5.1 inches, height 2.1 inches, and slant height 5.5 inches. Express your answer in terms of \( \pi \).

13. Find the surface area of a regular square pyramid. The base has an edge of length 12 cm and the slant height is \( 6\sqrt{10} \) cm. (exact value)

14. If the edges of a cube are tripled, what happens to the surface area? [Hint: try it using "x" as the original side length]

15. A semicircular cylinder is formed by cutting a solid cylinder with a radius of 8 ft. and height of 10 ft. in half along a diameter ("length-wise"). Find the surface area of the semicircular cylinder. (exact value)

16. Find the length of the edge of a cube whose surface area is 294 square units.
17. Find the height of a right cylinder whose surface area is \(4\pi\) square units and whose radius is 0.5 units.

18. A rectangular prism has a surface area of 136 square units. If the width of the prism is 4 units and the height is 3 units, what is the length of the prism?

19. Find the volume.

20. Find the volume in terms of \(\pi\).

21. If the surface area of a cube is 216, find the volume of the cube.

22. If the volume of a rectangular prism is expressed as \(x^3 - 2x^2 - 3x\), then the dimensions could be which of the following:

   (a) \(x, (x+1), (x+3)\)  
   (b) \((x^2 + 3), (x-1)\)  
   (c) \(x, (x+1), (x-3)\)  
   (d) \((x^2 - 3), (x+1)\)

23. Which has the same volume as a sphere with radius 6?

   (a) a cylinder with radius 4 and height 16
   (b) a cone with radius 6 and height 24
   (c) a cube whose edges are 6
   (d) a pyramid with base area 36 \(u^2\) and height 6

24. If the base edge of a square pyramid is doubled, what will happen to the volume of the pyramid?

   (a) it will be half  
   (b) it will double  
   (c) it will increase four times  
   (d) it will increase six times
25. If the circumference of the base of a cylinder is $2\pi$ and the height is 1, find the surface area. State the answer in terms of $\pi$.

26. The height of a cone is 9 cm and the radius of the base is 12 cm. Find the surface area of the cone. State the answer in terms of $\pi$.

Find the surface area and the volume for each figure. State the answer in terms of $\pi$ or in simplest radical form if necessary.

27. 

28. 

29. A regulation baseball must have a circumference between 9 inches and 9.25 inches. Find the range of baseball surface areas. Round your answers to the nearest tenth.

30. What is the height of a cement patio that has a length of 144 inches, a width of 168 inches, and was filled with 96,768 in$^3$ of cement?

31. What is the volume of the largest ball that will fit into a cubical box whose sides measure 8 inches? State the answer in terms of $\pi$.

32. If a cylindrical water tank which is 400 cm tall with a diameter of 200 cm is filled with water, how many conical shaped paper cups can be completely filled with water if the cups have a 12 cm diameter and are 20 cm tall? [Assume there is no waste of the water!]

33. One paint roller has length 11 inches and a diameter of 2 inches. A second roller has length 7 inches and a diameter of 3 inches. Which roller can spread more paint in one revolution? How much more paint (nearest tenth of a square inch)?
Find the surface area and volume for each figure. Leave answers in simplest radical form if necessary.

34. 

35. square pyramid

36. If the following tank is in the shape of a cylinder with ends that are hemispheres, find the total volume of the tank. State the answer in terms of $\pi$ and then also rounded to the nearest tenth.
1. A quadrilateral with sides of 8 cm, 9 cm, 6 cm, and 5 cm has area $45\;\text{cm}^2$. Find the area of a similar quadrilateral whose longest side is 15 cm.

2. A pentagon with sides of 3 m, 4 m, 5 m, 6 m, and 7 m has area $48\;\text{m}^2$. Find the perimeter of a similar pentagon whose area is $27\;\text{m}^2$.

3. Two spheres have diameters of 24 cm and 36 cm. Find the ratio of their surface areas and the ratio of their volumes.

4. Two spheres have volumes of $2\pi$ and $16\pi$ cubic units respectively. Find the ratio of their diameters and the ratio of their surface areas.

5. Two regular octagons have perimeter 16 cm and 32 cm, respectively. What is the ratio of their areas?

6. Two similar polygons have a scale factor 7:5. The area of the large polygon is 147. Find the area of the smaller polygon.

7. Two cones have radii of 6 cm and 9 cm. Their heights are 10 cm and 15 cm respectively. Are the cones similar? Find the ratio of their volumes.
8. Two similar pyramids have heights of 12 and 18. Find the ratio of their base areas and the ratio of their lateral areas.

9. The areas of two circles are $100\pi$ and $36\pi$. Find the ratio of their radii and the ratio of their circumferences.

10. Two regular pentagons have sides of 14 m and 3.5 m, respectively. Find their scale factor and the ratio of their areas.

11. The diagonal of one cube has a length of $2\sqrt{2}$ cm. A diagonal of another cube is $4\sqrt{2}$ cm. The larger cube has a volume of 64 cm$^3$. Find the volume of the smaller cube.

12. Two similar cones have radii of 4 cm and 6 cm. The total surface area of the smaller cone is $36\pi$ cm$^2$. Find the total surface area of the larger cone.

13. Two triangles are similar. The longest side of one triangle is five times as long as the corresponding side of the second triangle. How many times greater is the area of the larger triangle than the smaller triangle?

14. Two similar triangles have areas of 36 cm$^2$ and 16 cm$^2$. If a side of the larger triangle is 12 cm in length, find the length of the corresponding side in the smaller triangle.

15. Two similar cones have volumes of $8\pi$ and $27\pi$. Find the ratio of their surface areas.

16. Two similar pyramids have volumes of 9 and 729 respectively. Find the ratio of their heights and the ratio of their surface areas.
17. The area of one equilateral triangle is three times the area of a smaller equilateral triangle with a side of 8 cm. What is the length of a side of the larger triangle?

18. The ratio of the perimeters of two similar triangles is 2:3. If the area of the smaller triangle is 15 m², find the area of the larger triangle.

19. The length of each edge of one pyramid is five times the length of each edge of a similar pyramid. Find the ratio of their volumes.

20. The volume of the larger of two similar right rectangular prisms is 480 cm³. The heights of the prisms are 9 cm and 12 cm, respectively. Find the volume of the smaller prism.
Chapter 12 Review

In #1-10, name the solid. Find the exact surface area and volume.

1. \( \text{Cylinder with radius } 12 \text{ km and height } 10 \text{ km} \)

2. \( \text{Triangular Pyramid with base } 5 \text{ in.}, \text{ height } 4 \text{ in., and slant height } 3 \text{ in.}\)

3. \( \text{Cylinder with radius } 8 \text{ km and height } 10 \text{ km} \)

4. \( \text{Rectangular Pyramid with base } 20 \times 25 \text{ cm, height } 24 \text{ cm, and slant height } 26 \text{ cm} \)

5. \( \text{Rectangular Prism with } 10 \times 4 \times 7 \text{ ft.} \)

6. \( \text{Triangular Pyramid with base } 6 \text{ m}, \text{ height } 4 \text{ m, and slant height } 5 \text{ m} \)
In #11, find the exact surface area and volume.
12. The surface area of a rectangular prism with length 4 and width 8 is 136 square inches. Find the height of the prism.

13. The volume of a cube is 357.911 cubic meters. Find the length of a side of the cube.

14. What is the surface area of a cube if the area of one side of the cube is 144 in$^2$.

15. Find the surface area of a rectangular prism whose dimensions are 4 ft. x 6 ft. x 2 ft.

16. A triangular pyramid has a base that is congruent to its lateral faces. The base edge is 24 cm and the slant height is $12\sqrt{3}$ cm. What is the surface area of the pyramid?(exact & rounded nearest tenth)

17. The volume of a rectangular prism is 90 cm$^3$. If the length is 6 cm and the width is 3 cm, find the height of the prism.

18. A square pyramid has a volume of 400 ft$^3$. If the height is 13 ft, find the measure of a side of the base. (round nearest tenth)

19. Find the volume of a cube with a surface area of 96 m$^2$.

20. How does doubling the height and radius of a cylinder affect the volume?

21. A volcano has the shape of a cone. The diameter of the base of the volcano is about 9 miles. The height of the volcano is about 1000 feet. What is the approximate volume of the volcano to the nearest cubic mile? (Use 1 mile = 5280 feet)

22. Find the volume of a right triangular prism whose base is an isosceles triangle with legs measuring 4 inches and a base of 6 inches, and whose height is 13 inches. (nearest tenth)

23. A basketball has a radius of approximately 4.75 inches when filled. How much material is needed to make one? How much air will it hold? (nearest tenth)

24. A pile of sand has a circular base that is 8 yards in diameter. If the pile is $3\frac{1}{2}$ yards high, about how much sand is in the pile? (nearest tenth)

25. Find the height of a regular pyramid with volume 549.6 cm$^3$ and a base area of 44.2 cm$^2$. Round to the nearest tenth.
26. The volume of a rectangular pool is 15,360 cubic meters. Its length, width, and depth are in the ratio of 10:6:4. Find the number of meters in each of the three dimensions of the pool.

Chapter 12 Review - ANSWERS

1. sphere; $SA=144\pi \text{ mm}^2; V=288\pi \text{ mm}^3$
2. cone; $SA=24\pi \text{ in.}^2; V=12\pi \text{ in.}^3$
3. cylinder; $SA=112\pi \text{ km}^2; V=160\pi \text{ km}^3$
4. triangular prism; $SA=2280 \text{ cm}^2, V=6000 \text{ cm}^3$
5. rectangular prism; $SA=276 \text{ ft.}^2, V=280 \text{ ft.}^3$
6. square pyramid; $SA=96 \text{ m}^2, V=48 \text{ m}^3$
7. triangular prism; $SA=192 \text{ mi.}^2, V=144 \text{ mi.}^3$
8. triangular prism; $SA=117 \text{ m}^2, V=63 \text{ m}^3$
9. $SA=57\pi \text{ cm}^2; V=63\pi \text{ cm}^3$

10. $SA=284 + 91\pi \text{ km}^2; V=420 + 147\pi \text{ km}^3$
11. $SA=340 \text{ mi.}^2; V=372 \text{ mi.}^3$

12. $h = 3 \text{ in.}$
13. $s = 7.1 \text{ m}$
14. $SA = 864 \text{ sq. in.}$
15. $88 \text{ sq. ft.}$
16. $576\sqrt{3} \text{ cm}^2; 997.7 \text{ cm}^2$
17. $h = 5 \text{ cm}$
18. $S = 9.6 \text{ ft}$
19. $V = 64 \text{ m}^3$
20. Volume is multiplied by 8
21. $V = 4 \text{ mi}^3$
22. $103.2 \text{ in}^3$
23. $SA = 283.5 \text{ in}^3, V = 448.9 \text{ in}^3$
24. $V = 58.6 \text{ yds}^3$
25. $h = 37.3 \text{ cm}$
26. $40\text{ m}, 24\text{ m}, 16\text{ m}$