Transformations Packet
Geometry
# Chapter 14: Transformations

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| Chapter 12 | Extra Practice | Pg 597 (Self Test 1) # 1-18  
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Transformations Rules
(not given on reference sheet for exams)

Reflections: “r”

1. $r_{y-axis}(x, y) \rightarrow (\quad , \quad )$
2. $r_{origin}(x, y) \rightarrow (\quad , \quad )$

3. $r_{x-axis}(x, y) \rightarrow (\quad , \quad )$
4. $r_{y=-x}(x, y) \rightarrow (\quad , \quad )$

Translations: “T”

5. $T_{p,q}(x, y) \rightarrow (\quad , \quad )$

Rotations: “R”

6. $R_{90}(x, y) \rightarrow (\quad , \quad )$
7. $R_{180}(x, y) \rightarrow (\quad , \quad )$

8. $R_{270}(x, y) \rightarrow (\quad , \quad )$

Dilations: “D”

9. $D_{v}(x, y) \rightarrow (\quad , \quad )$
A **transformation** is an operation on a geometric figure that preserves a one-to-one correspondence between every point in the original figure and a new point on the transformed **image** of the figure.

There are four types of transformations: *translation, reflection, rotation, dilation*

**Prime Notation:**

A’ corresponds to A. A’ is called the image of A.

The **image** occurs after the transformation.

Similarly we say A is the **preimage** of A’ because A occurred before the transformation.

**Examples:**

![Translation example](image1)

**The Four Types of Transformations:**

1. **Translation:** Create a new image of a geometric figure by sliding it left, right, up, or down to a new location.

![Translation example](image2)
Notation: \[ T_{a,b} \ P(x,y) \rightarrow P'(x+a, y+b) \]

This translates any point a unit right and b units up. For example, a translation of \( T_{1,-2} \) on the point \( P(-3, 2) \) would be translated to \( (-3+1, 2-2) \) or \( P'(-2, 0) \).

\[ T_{1,-2} \ P(-3,2) \rightarrow P'(-3+1,2+(-2)) \]
\[ SO, \]
\[ T_{1,-2} \ P(-3,2) \rightarrow P'(-2,0) \]

Practice:
(a) Write a rule for the transformation performed if \( Q(3, 5) \) translates to \( Q'(3, 18) \).

(b) Consider the point \( Q(4,-3) \) which is translated to \( (3,2) \). What would the preimage of \( P(1, 2) \) be under the same translation?

2. Reflection: create a mirror image of a geometric figure.

Two types of reflections:
(a) A line reflection where every image is the same distance to the line as the preimage. Notice the line is a perpendicular bisector.

Reflection of a triangle across the line \( y=x \)
(b) A point reflection where every image is the same distance to the point as the preimage.

Notice \( A \) to \( A' \). It goes through the point (in this case the origin) and continues the same amount of distance.

\[
\begin{align*}
  r_{\text{origin}} P(x, y) & \rightarrow P'(-x, -y) \\
  r_{x\text{-axis}} P(x, y) & \rightarrow P'(x, -y)
\end{align*}
\]

Notation:
\[
\begin{align*}
  r_{y\text{-axis}} P(x, y) & \rightarrow P'(-x, y) \\
  r_{y=x} P(x, y) & \rightarrow P'(y, x)
\end{align*}
\]

3. Rotations: Create a new image of a geometric figure by rotating it clockwise or counter-clockwise about a fixed point. Think Pinwheel!!

Notation: \( R_{\text{origin, 90}} \) (\( R \) representing rotation, the first being the point we are revolving around, the second being the number of degrees we are revolving in a clockwise or counterclockwise manner).

For shorthand, if no point is given, it is the origin, by default, we revolve around.

\[
\begin{align*}
  R_{90} P(x, y) & \rightarrow P'(-y, x) \\
  R_{180} P(x, y) & \rightarrow P'(-x, -y) \\
  R_{270} P(x, y) & \rightarrow P'(y, -x)
\end{align*}
\]

Rotate counterclockwise when rotating _____________ degrees.

Rotate clockwise when rotating _____________ degrees.
Question:
(a) In the picture, if we revolve Triangle A around the origin 90 degrees, what triangle will we get?

(b) What would be the single transformation if D is our image and A is the preimage?

Answer: \( R\_\_\_\_\_\_ \) or \( R\_\_\_\_\_\_ \)

4. **Dilations**: create a larger or a smaller image of a geometric figure. The amount in increase or decrease in the size of a geometric figure is called the scalar, the scale factor, or the constant of dilation. All points are multiplied by this scale factor, typically denoted using \( k \).

The original figure gets \textbf{larger} when the scalar is \textbf{greater than 1}. The original figure gets \textbf{smaller} when the scalar is \textbf{between zero and 1} (or a fraction).

Note: If \( k \) is negative, it also \textit{reflects} the image through the origin.

Examples:
**Notation:** $D_o, k$

The D stands for dilation, $O$ represents the point which we are dilating through and $k$ is the scale factor.

Note: If no point is given which we are dilating through, it is, by default, the origin.

Note: Dilation is the only transformation where the image and preimage are not congruent (unless $k = 1$ or $-1$).

**Properties of Transformations**

**Isometry** - An isometry literally means “same measure”. An isometry occurs when the original geometric figure and its transformed image are congruent. Thus, distances are preserved.

Which transformations are isometries?

Which transformations are NOT isometries?

**Orientation** is preserved when the transformed image is “facing the same direction” - the top is still “up” and the shape is “not turned” in any other direction from the original shape. The vertices of the shape also have the same direction (clockwise or counterclockwise).

**Indirect** (also called opposite) vs. **Direct Isometries**

**Direct Isometries** are transformations that not only preserve distance but also preserve orientation whereas an **Indirect Isometry** does not preserve orientation.

Example:

An example of a direct isometry is a translation. Notice in the picture that if you draw a circle from $A$ to $B$ to $C$ that it is in the same direction as if you draw a circle from $A'$ to $B'$ to $C'$.
Example:
An example of an indirect (opposite) isometry is a line reflection. Notice in the picture that if you draw a circle from A to B to C that it is in the opposite direction as if you draw a circle from A' to B' to C'. Another way to think of this is a mirror. When you look in a mirror and wave your right hand, the image waves the opposite or left hand.

NOTES:
Translations preserve shape, size, and orientation. Hence, translations are isometries of their original geometric figures with the same orientations. Therefore, translations are _____________ Isometries.

Reflections preserve shape and size, but not orientation. Hence, reflected images are isometries of their original geometric figures with opposite orientations. Therefore, reflections are ______________ Isometries.

Rotations preserve shape, size, and orientation. Hence, rotations are isometries of their original geometric figures with the same orientations. Therefore, rotations are ______________ Isometries.

Dilations preserve shape and orientation, but not size. Hence, dilated images are similar to their original geometric figures, but they are not _________________.

Symmetry

Symmetry - If a transformation is performed on an image and the result is the same image, then the object is said to have symmetry.

Types of symmetry with examples-

Line symmetry - If a line can be drawn on an image such that it "folds" on top of itself, the object has line symmetry.

Ex. Letters- ________________ (many more) have line symmetry
Letters- ________________ do not have line symmetry.
Letters- ________________ have more than one line that creates symmetry
Polygons with line symmetry include Squares, Rectangles, and any Regular Polygon.

![Image of polygons with line symmetry]

Polygon without line symmetry include parallelograms, right trapezoids and scalene triangles.

**Point Symmetry** - If the image is reflected through a point onto itself, it is said to have point symmetry. The easiest way to tell is by turning the object upside down. If the object is the same as the preimage, then the object has point symmetry.

Letters- ______________ have point symmetry
Letters- ______________ do not have point symmetry

Polygons with point symmetry include parallelograms and any even numbered polygon (look at the regular polygons and notice that when we turn the paper upside down, the point of the pentagon has changed from pointing up to pointing down).

**Rotational symmetry** - Rotational symmetry occurs when we can turn the image around its center a certain number of degrees and the result will be the same object.

*Every object has rotational symmetry of 360° but we consider this case trivial since all of the figures have it.

*Every figure that has point symmetry will also have rotational symmetry of 180°.  
*Other interesting cases include the circle which can be turned any number of degrees to result in the same circle.  
*Regular polygons also have rotational symmetry. To calculate the number of degrees we need to rotate the figure we just do ______.

Example:
What is the minimum amount that we need to rotate a regular hexagon in order to result in the same hexagon?

**Answer:**
**Practice**

1. Triangle: A(2, 4), B(-1, 3), C(2, -3)  
   Translation: 4 units to the right  
   Reflection: over y-axis

   A'(_____, ____)
   B'(_____, ____)
   C'(_____, ____)

   Rule: T_____ (x, y) \rightarrow (_____ , _____)

2. Triangle: A(1, 3), B(-2, 6), C(0, 0)  
   Translation: 4 units to the right  
   Reflection: over y-axis

   A'(_____, ____)
   B'(_____, ____)
   C'(_____, ____)

   Rule: r_y-axis (x, y) \rightarrow (_____ , _____)

3. Triangle: A(1, 1), B(-3, 1), C(-5, -4)  
   Rotation: 180°

   A'(_____, ____)
   B'(_____, ____)
   C'(_____, ____)

   Rule: R_{180}(x, y) = (_____ , _____)

4. Triangle: A(2, 2), B(3, -5), C(-1, 0)  
   Rotation: 180°  
   Reflection: across x-axis

   A'(_____, ____)
   B'(_____, ____)
   C'(_____, ____)

   Rule: r_{x-axis}(x, y) = (_____ , _____)
5. Triangle: A(0, 0), B(5, 2), C(4, 4)
   Reflection: over line y = x
   \[ A'(_____, _____) \]
   \[ B'(_____, _____) \]
   \[ C'(_____, _____) \]
   Rule: \[ r_{y=x}(x, y) = (_____, _____) \]

Describe the transformation that is represented by the given rule.

6. \[ F(x, y) = (x + 3, y) \]

7. \[ T(x, y) = (-x, y) \]

8. \[ A(x, y) = (x - 2, y + 4) \]

9. \[ Q(x, y) = (x, y + 2) \]

10. \[ K(x, y) = (x, y - 5) \]

11. \[ H(x, y) = (-x, -y) \]

12. \[ B(x, y) = (x, -y) \]

13. \[ D(x, y) = (x, -y) \]

14. Which of the following transformations creates a figure that is similar (but not congruent) to the original figure?  
   I. translation  II. rotation  III. Dilation
   [A] II only  [B] I only  [C] II and III  [D] I and II  [E] III only

15. Is the following transformation a translation or rotation? Justify your answer.

16. Describe two different isometries under which \( \triangle DEF \) is an image of \( \triangle ABC \).
17. Which of the following shows a triangle and its translated image?

[A]  

[B]  

[C]  

[D]  

18. Which transformation is illustrated by the accompanying diagram?

(1) translation  (3) rotation  

(2) reflection  (4) dilation  

19. Which transformation produces a figure that is always the mirror image of the original figure?

(1) line reflection  (3) translation  (2) dilation  (4) rotation  

20. Which transformation does not always result in an image that is congruent to the original figure?

(1) dilation  (3) rotation  (2) reflection  (4) translation  

21. In the accompanying diagram, \( \triangle A'B'C' \) is the image of \( \triangle ABC \) and \( \triangle A'B'C' \cong \triangle ABC \).

Which type of transformation is shown in the diagram?

(1) line reflection  (3) translation  

(2) rotation  (4) dilation  

22. The accompanying diagram shows the transformation of \( \triangle XYZ \) to \( \triangle X'Y'Z' \). This transformation is an example of a

(1) line reflection  (3) translation  

(2) rotation  (4) dilation  

23. Which type of transformation is illustrated in the accompanying diagram?

(1) dilation  (3) translation  

(2) reflection  (4) rotation
24. A picture held by a magnet to a refrigerator slides to the bottom of the refrigerator, as shown in the accompanying diagram.

This change of position is an example of a
(1) translation (3) rotation
(2) dilation (4) reflection

25. The perimeter of \( \triangle A'B'C' \), the image of \( \triangle ABC \), is twice as large as the perimeter of \( \triangle ABC \). What type of transformation has taken place?
(1) dilation (3) rotation (2) translation (4) reflection

26. The accompanying diagram shows a transformation.

Which transformation performed on figure 1 resulted in figure 2?
(1) rotation (3) dilation
(2) reflection (4) translation

27. In the accompanying diagram, which transformation changes the solid-line parabola to the dotted-line parabola?
(1) translation (3) rotation, only
(2) line reflection, only (4) line reflection or rotation

28. Point \( P' \) is the image of point \( P(-3,4) \) after a translation defined by \( T_{(7,-1)} \). Which other transformation on \( P \) would also produce \( P' \)?
(1) \( r_y = x \) (2) \( R_{90^\circ} \) (3) \( r_y \text{--axis} \) (4) \( R_{-90^\circ} \)

29. Which transformation does not always produce an image that is congruent to the original figure?
(1) translation (2) rotation (3) dilation (4) reflection

30. Which transformation(s) preserve the shape and size of the object?

31. Which transformation(s) do not preserve the shape and size of the object?
32. Which transformation(s) preserve the orientation of the object?

33. Which transformation(s) do not preserve the orientation of the object?

34. Circle whether an isosceles trapezoid has reflectional symmetry, rotational symmetry, both kinds of symmetry, or neither kind of symmetry.
   If the trapezoid has reflectional symmetry, how many lines of symmetry?
   If the trapezoid has rotational symmetry, how many degrees of rotational symmetry?

35. Circle whether a regular decagon has reflectional symmetry, rotational symmetry, both kinds of symmetry, or neither kind of symmetry.
   If the regular decagon has reflectional symmetry, how many lines of symmetry?
   If the regular decagon has rotational symmetry, how many degrees of rotational symmetry?

36. Name a letter of the alphabet with only vertical line symmetry.

37. Name a letter of the alphabet with only horizontal line symmetry.

38. Name a letter of the alphabet with both horizontal and vertical line symmetry.

39. Name a quadrilateral with point symmetry.

40. Name all quadrilaterals with 90 degree rotational symmetry.

41. Name a polygon with 60 degree rotational symmetry.

42. Which figure has no point symmetry?
   (1) square  (2) regular hexagon  (3) rhombus  (4) isosceles triangle
Transformations Homework #1

1. Triangle $A'B'C'$ is the image of $\triangle ABC$ under a dilation such that $A'B' = 3AB$. Triangles $ABC$ and $A'B'C'$ are
   (1) congruent but not similar
   (2) similar but not congruent
   (3) both congruent and similar
   (4) neither congruent nor similar

2. What is the image of point $(-3,-1)$ under a reflection in the origin?
   (1) $(3,1)$  (3) $(1,3)$  (2) $(-3,1)$  (4) $(-1,-3)$

3. Under which transformation is the area of a triangle not equal to the area of its image?
   (1) rotation  (3) line reflection  (2) dilation  (4) translation

4. What is the image of $(x,y)$ after a translation of 3 units right and 7 units down?
   (1) $(x+3,y-7)$  (3) $(x-3,y-7)$  (2) $(x+3,y+7)$  (4) $(x-3,y+7)$

5. A polygon is transformed according to the rule: $(x,y) \rightarrow (x+2,y)$. Every point of the polygon moves two units in which direction?
   (1) up  (3) left  (2) down  (4) right

6. Which transformation is not an example of an isometry?
   (1) line reflection  (3) translation  (2) rotation  (4) dilation

7. What is the image of point $(2,5)$ under the translation that shifts $(x,y)$ to $(x+3,y-2)$?
   (1) $(0,3)$  (3) $(5,3)$  (2) $(0,8)$  (4) $(5,8)$

8. What are the coordinates of $P'$, the image of $P(-4,0)$ under the translation $(x-3,y+6)$?
   (1) $(-7,6)$  (3) $(1,6)$  (2) $(7,-6)$  (4) $(2,-3)$

9. Ms. Brewer's art class is drawing reflected images. She wants her students to draw images reflected in a line. Which diagram represents a correctly drawn image?

10. Under the transformation $(x,y) \rightarrow (2x,2y)$, which property is not preserved?
    (1) distance  (2) parallelism  (3) orientation  (4) angle measure
11. Which transformation represents a dilation?
(1) (8,4) → (11,7)  (2) (8,4) → (−4, −8)  (3) (8,4) → (−8,4)  (4) (8,4) → (4,2)

12. Which transformation does not preserve orientation?
(1) \( T_{3,5} \)  (2) \( D_4 \)  (3) \( r_{y=x} \)  (4) \( R_{360} \)

13. In which quadrant would the image of point (5,−3) lie after a dilation using a factor of -3?
(1) I  (2) II  (3) III  (4) IV

14. The image of point (3,−5) under the translation that shifts \((x,y)\) to \((x-1, y-3)\) is
(1) (-4,8)  (2) (-3,15)  (3) (2,8)  (4) (2,-8)

15. Under a dilation where the center of dilation is the origin, the image of \( A(-2, -3) \) is \( A'(-6, -9) \). What are the coordinates of \( B' \), the image of \( B(4,0) \) under the same dilation?
(1) (-12,0)  (2) (12,0)  (3) (-4,0)  (4) (4,0)

16. The image of point \( A \) after a dilation of 3 is (6,15). What was the original location of point \( A \)?
(1) (2,5)  (2) (3,12)  (3) (9,18)  (4) (18,45)

17. Which image represents a line reflection?
(1) \( P \)
(2) \( \overline{P} \)
(3) \( \overline{P} \)
(4) \( \overline{P} \)

18. Under a dilation with respect to the origin, the image of \( P(-15,6) \) is \( P'(−5,2) \). What is the constant of dilation?
(1) -4  (2) \( \frac{1}{3} \)  (3) 3  (4) 10

19. Write an equation of the line of reflection that maps \( A(1,8) \) onto \( A'(8,1) \).

20. Transformation \( D_k \) maps (-3, 6) to (-1, 2). What is the value of \( k \)? What is the image of (-6, -12) under the same transformation?

21. Find the image of (-2, 6) under a reflection in the origin.
More Practice

1. On the accompanying set of axes, graph \( \triangle ABC \) with coordinates \( A(-1,2) \), \( B(0,6) \), and \( C(5,4) \). Then graph \( \triangle A'B'C' \), the image of \( \triangle ABC \) after a dilation of 2.

2. When the point \( (2,-5) \) is reflected in the \( x \)-axis, what are the coordinates of its image?
   (1) \((-5,2)\)  (3) \((2,5)\)  (2) \((-2,5)\)  (4) \((5,2)\)

3. In the accompanying figure, point \( P \) is the center of the square. Find the image of each of the indicated letters under the given notation.
   a) \( R_{P,90}(W) \)  b) \( R_{P,180}(Z) \)  
   c) \( R_{P,270}(X) \)  d) \( R_{P,-90}(Y) \)

4. On the accompanying grid, graph and label quadrilateral \( ABCD \), whose coordinates are \( A(-1,3) \), \( B(2,0) \), \( C(2,-1) \), and \( D(-3,-1) \). Graph, label, and state the coordinates of \( A'B'C'D' \), the image of \( ABCD \) under a dilation of 2, where the center of dilation is the origin.
5. In the graph below, the shaded region represents set \( A \) of all points \((x,y)\) such that \( x^2 + y^2 \leq 1 \). The transformation \( T \) maps point \((x, y)\) to point \((2x, 4y)\). Which graph to the right shows the mapping of set \( A \) by the transformation \( T \)?

6. Matthew is a fan of the Air Force’s Thunderbirds flying team and is designing a jacket patch for the team, as shown in the accompanying diagram. If \( P \) has the coordinates \((a, b)\), what are the coordinates of \( Q \), the reflection of \( P \) in the line \( y = x \)?

   (1) \((a, b)\)  (2) \((b, a)\)  (3) \((-a, b)\)  (4) \((y, x)\)

7. A design was constructed by using two rectangles \( ABDC \) and \( A'B'D'C' \). Rectangle \( A'B'D'C' \) is the result of a translation of rectangle \( ABDC \). The table of translations is shown below. Find the coordinates of points \( B \) and \( D' \).

<table>
<thead>
<tr>
<th>Rectangle ( ABDC )</th>
<th>Rectangle ( A'B'D'C' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (2,4)</td>
<td>A' (3,1)</td>
</tr>
<tr>
<td>B</td>
<td>B' (-5,1)</td>
</tr>
<tr>
<td>C (2,-1)</td>
<td>C' (3,-4)</td>
</tr>
<tr>
<td>D (-6,-1)</td>
<td>D'</td>
</tr>
</tbody>
</table>

8. In the accompanying graph, if point \( P \) has coordinates \((a, b)\), which point has coordinates \((-b, a)\)?

   (1) \( A \)  (2) \( B \)  (3) \( C \)  (4) \( D \)

9. In the accompanying diagram of square \( ABCD \), \( F \) is the midpoint of \( \overline{AB} \), \( G \) is the midpoint of \( \overline{BC} \), \( H \) is the midpoint of \( \overline{CD} \), and \( E \) is the midpoint of \( \overline{DA} \). Find the image of \( \triangle EOA \) after it is reflected in line \( \ell \). Is this isometry direct or opposite? Explain your answer.
10. If \( x = -3 \) and \( y = 2 \), which point on the accompanying graph represents \((-x,-y)\)?

(1) \( P \)  \( \quad \) (3) \( R \)
(2) \( Q \)  \( \quad \) (4) \( S \)

11. If point \((5,2)\) is rotated counterclockwise \(90^\circ\) about the origin, its image will be point

(1) \((2, 5)\)  \( \quad \) (3) \((-2, 5)\)  \( \quad \) (2) \((2, -5)\)  \( \quad \) (4) \((-5, -2)\)

12. Carson is a decorator. He often sketches his room designs on the coordinate plane. He has graphed a square table on his grid so that its corners are at the coordinates \( A(2,6) \), \( B(7,8) \), \( C(9,3) \), and \( D(4,1) \). To graph a second identical table, he reflects \( ABCD \) over the \( y\)-axis. On the accompanying set of coordinate axes, sketch and label \( ABCD \) and its image \( A'B'C'D' \), which show the locations of the two tables. Then find the number of square units in the area of \( ABCD \).

13. If \( x = -2 \) and \( y = -1 \), which point on the accompanying set of axes represents the translation \((x,y) \rightarrow (x + 2, y - 3)\)?

(1) \( Q \)  \( \quad \) (3) \( S \)
(2) \( R \)  \( \quad \) (4) \( T \)

14. The image of point \((-2,3)\) under translation \( T \) is \((3,-1)\). What is the image of point \((4,2)\) under the same translation?

(1) \((-1,6)\)  \( \quad \) (3) \((5,4)\)  \( \quad \) (2) \((0,7)\)  \( \quad \) (4) \((9,-2)\)
15. The accompanying diagram shows the starting position of the spinner on a board game.

How does this spinner appear after a 270° counterclockwise rotation about point P?

16. Two parabolic arches are to be built. The equation of the first arch can be expressed as $y = -x^2 + 9$, with a range of $0 \leq y \leq 9$, and the second arch is created by the transformation $T_{7,0}$.

On the accompanying set of axes, graph the equations of the two arches. Graph the line of symmetry formed by the parabola and its transformation and label it with the proper equation.

17. Which of the following rotations about the origin is not equivalent to the other three?
   (1) 90 counterclockwise  (3) -270  (2) 270 clockwise  (4) -90

18. Which transformation is not an isometry?
   (1) $(x, y) \rightarrow (x + 6, y - 2)$  (3) $(x, y) \rightarrow (y, -x)$
   (2) $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$  (4) $(x, y) \rightarrow (-y, -x)$

19. Which property is not preserved by a glide reflection?
   (1) betweeness  (3) orientation  (2) angle measure  (4) collinearity
20. A line reflection preserves
(1) distance and orientation
(2) angle measurement and orientation
(3) distance, but not angle measurement
(4) distance and angle measurement

21. Given: Triangle ABC with A(2, 3), B(0, 6), and C(2,6).
Graph and state the coordinates of:

a. \( \triangle A'B'C' \), the image of \( \triangle ABC \) after \( r_{y\text{-axis}} \).
b. \( \triangle A''B''C'' \), the image of \( \triangle A'B'C' \) after \( r_{y=x} \).
c. \( \triangle A'''B'''C''' \), the image of \( \triangle A''B''C'' \) after \( R_{0,90} \).
Section 14-3 (glide reflections) and 14-8: Composite Transformations and Glide Reflections

**Composite Transformations** - The process of using the image of one transformation as the coordinate to be used in another transformation.

- The order in which the geometric transformations are composed matters.

1. Find the coordinates of the image of \( P(-2, 4) \) under each composite transformation:
   (a) \( r_{y=x} \circ r_{y-axis} (-2,4) = \)

   (b) \( r_{y-axis} \circ r_{y=x} (-2,4) = \)

2. In the accompanying figure, \( p \) and \( q \) are lines of symmetry for regular hexagon ABCDEF intersecting at point \( O \), the center of the hexagon. Determine the image of each composite transformation:
   (a) \( r_p \circ r_q (AB) = \)

   (b) \( r_q \circ r_p \circ r_q (D) = \)

3. In the accompanying diagram, regular hexagon ABCDEF is inscribed in circle \( O \). Line \( p \) is a line of symmetry. What is the image of \( r_p \circ R_{240^\circ} (F) \)?

It will always be assumed that the center of rotation is the origin and that a positive angle of rotation turns a figure in the counterclockwise direction. A negative angle of rotation turns a figure in the clockwise direction.
**Glide Reflection** - a composite transformation in which a figure is reflected in a line and then the image is translated parallel to the reflecting line. In the figure (below right), $\triangle A'B'C'$ is the image of $\triangle ABC$ under a glide reflection that reflects $\triangle ABC$ in line $l$ and translates its image, $\triangle A'B'C'$, a fixed distance parallel to line $l$.

Another Example: Footprints in the sand (above left)

Examples:
1. Find the image of $\triangle ABC$ under the glide reflection $(-2, 0)$ and $y = -1$.
2. Find the image of $\triangle ABC$ under the glide reflection $(0, -6)$ and $x = 3$. 
PRACTICE

Multiple Choice.

1. Which transformation is not an isometry?
   (1) \((x, y) \rightarrow (x + 2, y + 2)\)
   (2) \((x, y) \rightarrow (2x, 2y)\)
   (3) \((x, y) \rightarrow (y, -x)\)
   (4) \((x, y) \rightarrow (-x, -y)\)

2. Which transformation is an opposite isometry?
   (1) \((x, y) \rightarrow (x - 2, y - 2)\)
   (2) \((x, y) \rightarrow (-y, x)\)
   (3) \((x, y) \rightarrow (y, x)\)
   (4) \((x, y) \rightarrow (y, -x)\)

3. For any point \((x, y)\), which transformation is equivalent to \(R_{45} \circ R_{-135}\)?
   (1) \(R_{-90}\)
   (2) \(R_{90}\)
   (3) \(r_{y=x}\)
   (4) \(r_{x\text{-axis}}\)

4. Which composite transformation does not represent a glide reflection?
   (1) \(T_{4,0} \circ r_{x\text{-axis}}\)
   (2) \(T_{2,4} \circ r_{y=x}\)
   (3) \(T_{0,4} \circ r_{y\text{-axis}}\)
   (4) \(r_{x=1} \circ r_{y=3} \circ r_{x=5}\)

#5-7, In the accompanying diagram, \(l\) and \(m\) are lines of symmetry.

5. What is \(r_{m} \circ r_{l}(FG)\)?
   (1) \(CD\)
   (2) \(AH\)
   (3) \(HG\)
   (4) \(BC\)

6. What is \(r_{l} \circ r_{m}(AB)\)?
   (1) \(CD\)
   (2) \(AH\)
   (3) \(HG\)
   (4) \(BC\)

7. What is \(r_{m} \circ r_{l} \circ r_{m}(H)\)?
   (1) \(A\)
   (2) \(E\)
   (3) \(F\)
   (4) \(G\)
8. In the accompanying diagram, p and q are lines of symmetry for figure ABCDEF, what is 
\( r_p \circ r_q \circ r_p(A) \)?

(1) B   (1) D   (3) E   (4) F

#9-11. In the accompanying diagram, \( l \) is the line \( x = 2 \) and \( m \) is the line \( y = x \).

9. What are the coordinates of the image of \( r_l \circ r_m(P) \)?

(1) (3, 5)   (2) (-1, 1)   (3) (3, -5)   (4) (1, -1)

10. What are the coordinates of the image of \( r_m \circ r_l(P) \)?

(1) (-3, 5)   (2) (-1, 1)   (3) (3, -5)   (4) (1, -1)

11. What are the coordinates of the image of \( r_{\text{origin}} \circ r_l \circ r_{\text{x-axis}}(P) \)?

(1) (-1, -1)   (2) (1, 1)   (3) (3, 5)   (4) (-5, -3)

12. Given these transformations:
\( R(x, y) \rightarrow (-x, y) \) and \( S(x, y) \rightarrow (y, x) \)

What is \( (R \circ S)(5, -1) \)?

(1) (1, 5)   (2) (1, -5)   (3) (-1, 5)   (4) (-1, -5)

13. Which transformation does not have the same image as
\( r_{\text{x-axis}} \circ r_{\text{y-axis}} \ P(a, b) \), where \( a, b \neq 0 \)?

(1) \( r_{\text{x-axis}} \circ r_{\text{y-axis}} \ P(b, a) \)   (2) \( r_{\text{y-axis}} \circ r_{\text{x-axis}} \ P(a, b) \)

(2) \( r_{\text{origin}} \ P(a, b) \)   (4) \( r_{\text{x=y}} \ P(-b, -a) \)

14. The composite transformation that reflects point \( P \) through the origin, the \( x \)-axis, and the line \( y = x \), in the order given, is equivalent to which rotation?

(1) \( R_{90} \)   (2) \( R_{180} \)   (3) \( R_{270} \)   (4) \( R_{360} \)
In each case, show how you arrived at your answer by clearly indicating all of the necessary steps, formula substitutions, diagrams, graphs, charts, etc.

15. If \((-4, 8)\) is the image under the composite transformation \(T_{h,3} \circ T_{-2,k}(-3,0)\), what are the coordinates of the image of \((2, -1)\) under the same composite transformation?

16. (a) On graph paper, graph and label the triangle whose vertices are \(A(0,0)\), \(B(8,1)\), and \(C(8,4)\). Then graph and state the coordinates of \(\triangle A'B'C'\) under the composite transformation \(r_{x=0} \circ r_{y=x}(\triangle ABC)\).

(b) Which single type of transformation maps \(\triangle ABC\) onto \(\triangle A'B'C'\)?
   
   (1) rotation  (2) dilation  (3) glide reflection  (4) translation

17. (a) On graph paper, graph and label the triangle whose vertices are \(A(0,0)\), \(B(8,1)\), and \(C(8,4)\). Then graph and state the coordinates of \(\triangle A'B'C'\) under the composite transformation \(r_{y=-4} \circ r_{y=0}(\triangle ABC)\).

(b) Which single type of transformation maps \(\triangle ABC\) onto \(\triangle A'B'C'\)?
   
   (1) rotation  (2) dilation  (3) glide reflection  (4) translation

18. In the accompanying diagram, \(P\) is a line of symmetry of regular hexagon \(ABCDEF\). Name the point that is the image under each transformation:

   (a) \(R_{120} \circ r_p(C)\)
   
   (b) \(r_p \circ R_{-240}(B)\)
   
   (c) \(R_{120} \circ r_p \circ R_{-60}(A)\)
   
   (d) \(r_p \circ R_{-240} \circ r_p(F)\)
19. Farmington, NY, has plans for a new triangular park. If plotted on a coordinate grid, the vertices would be A(3, 3), B(5, -2), and C(-3, -1). However, a tract of land has become available that would enable the planners to increase the size of the park, which is based on the following transformation of the original triangular park, $R_{270} \circ D_2$. On the grid below, graph and label both the original park $\triangle ABC$ and its image, the new park $\triangle A''B''C''$, following the transformation.

20. If Q is (-4, 2), what is $r_{x-axis} \circ D_2(Q)$?

21. Find the coordinates of $B'$, the image of the point B(2, -3) under the transformation $r_{y=2} \circ r_{y-axis}$.

22. Write a single transformation that is equivalent to $T_{2,1} \circ T_{-3,2}$.
More Practice of Composite Transformations

1. If the coordinates of point $P$ are $(2, -3)$, then $R_{90} \circ R_{180}(P)$ is:
   a) $(-2, -3)$  b) $(-3, -2)$  c) $(3, -2)$  d) $(-2, 3)$

2. Given points $A(3, 0)$ and $B(-4, 6)$
   Write the coordinates of the images of $A$ and $B$ after each transformation described:
   a) the images of $A$ and $B$ after $r_{y=x}$
   b) the images of $A$ and $B$ after $R_{90}$
   c) the images of $A$ and $B$ after $r_{x=2}$
   d) the images of $A$ and $B$ after $r_{\text{origin}}$
   e) the images of $A$ and $B$ after $D_{\frac{1}{2}}$

3. What is the image of $P(-4, 6)$ under the composite $r_{x=2} \circ r_{y=\text{axis}}$?
   a) $(6, 0)$  b) $(0, 6)$  c) $(4, -2)$  d) $(-8, 6)$

4. The coordinates of $\overline{AB}$ are $A(5, 2)$ and $B(6, 5)$.
   a) Find the slope of $\overline{AB}$ and the equation of the line which contains $\overline{AB}$.
   b) On the same set of axes, graph and label $\overline{AB}$, $\overline{A'B'}$, and $\overline{A''B''}$ for the following composite transformation: $r_{y=x} \circ T_{6,-6}(\overline{AB})$

5. Triangle $ABC$ has coordinates $A(1, 2)$, $B(0, 5)$, and $C(5, 4)$.
   a) On graph paper, graph $\triangle ABC$
   b) Graph and state the coordinates of $\triangle A'B'C'$, $\triangle A''B''C''$, and $\triangle A'''B'''C'''$, which are the images of $\triangle ABC$ under the composite transformation: $r_{\text{origin}} \circ r_{x=\text{axis}} \circ T_{6,3}$

6. Triangle $ABC$ has vertices $A(1, 0)$, $B(6, 3)$, and $C(4, 5)$.
   a) Graph $\triangle ABC$
   b) Graph and state the coordinates of $\triangle A'B'C'$, and $\triangle A''B''C''$, which are the images of $\triangle ABC$ under the composite transformation: $D_2 \circ r_{(0,0)}$
7. Which transformation is not an isometry?
   a) rotation  b) dilation  c) translation  d) reflection

8. Triangle ABC has coordinates A(1, 0), B(7, 4), and C(5, 7).
   a) Graph \( \triangle ABC \)
   b) Graph and state the coordinates of \( \triangle A'B'C' \), and \( \triangle A''B''C'' \), which are the images of \( \triangle ABC \) under the composite transformation: \( T_{1,5} \circ r_{\text{origin}} \)
   c) Write an equation of the line which contains \( \overline{A''B''} \)

9. Triangle ABC has coordinates A(3, 4), B(1, 7), and C(3, 7).
   a) On graph paper, graph \( \triangle ABC \)
   b) Graph and state the coordinates of \( \triangle A'B'C' \), \( \triangle A''B''C'' \), and \( \triangle A'''B'''C''' \), which are the images of \( \triangle ABC \) under the composite transformation: \( T_{5,-1} \circ r_{y=x} \circ r_{y-\text{axis}} \)

10. The coordinates of point A are (3, -1). What are the coordinates of A under the transformation \( T_{2,5} \circ r_{x-\text{axis}} \)?
    a) (5, 6)  b) (5, 4)  c) (-5, -4)  d) (-1, 4)

11. The vertices of triangle ABC are A(-3, -2), B(2, 3), and C(5, -4).
    a) On graph paper, graph \( \triangle ABC \)
    b) Graph and state the coordinates of \( \triangle A'B'C' \), the image of \( \triangle ABC \) after \( D_2 \)
    c) Find the area of \( \triangle A'B'C' \)
Your Mathematical “Coat of Arms” – Perfect Score Assignment [Optional]

Due: Tuesday, May 29th (B/D Blocks)
Wednesday, May 30th (A/C Blocks)

[This is an all or nothing assignment, showing ALL CORRECT work will earn you
10 points]

You will design a mathematical “Coat of Arms”. A coat of arms is a symbol, usually on a shield, that represents your family. You will use your last name for this design. If your last name is less than 7 letters, you must add on your first name (no more than 12 letters-don’t go beyond the chart provided below). For example if your last name is SMITH, you would do SMITHJOHN. To build the design, you will use the coding system below.

0 1 2 3 4 5 6 7 8 9 10
A B C D E F G H I J K
L M N O P Q R S T U V
W X Y Z

Follow the example to code your name:

R I \rightarrow (6,8) See the first 2 columns in the table below. Write your name vertically in the first column, ending with your first letter a second time at the end of the column. The second column continues the name.
I C \rightarrow (8,2)
C K \rightarrow (2,10)
K E \rightarrow (10,4)
E R \rightarrow (4,6)
R T \rightarrow (6,8)
T R \rightarrow (8,6)
R I \rightarrow (6,8)

For the points, use the code above to form ordered pairs that represent your name.

Your Name:

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Now create the two different “Coat of Arms” on separate pieces of graph paper…..

**First Design:**

Graph your original points by graphing 1 at a time and connecting each point to the next *as you go along*. Then transform the original points by the reflection indicated and start graphing the new table of points with the first point and connecting in order again. Do the transformation for each table below.

<table>
<thead>
<tr>
<th>original points from name</th>
<th>original points reflected in x-axis</th>
<th>original points reflected in y-axis</th>
<th>original points reflected in origin</th>
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Color your “Coat of Arms” when you finish 😊
**Second Design:**
Graph your original points by graphing 1 at a time and connecting each point to the next *as you go along*. Then transform the original points by the reflection indicated and start graphing the new table of points with the first point and connecting in order again. Do the transformation for each table below.

<table>
<thead>
<tr>
<th>Original points from name</th>
<th>Original points rotated 90°</th>
<th>Original points rotated 180°</th>
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Color your “Coat of Arms” when you finish ☺️